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ROLE OF DIQUARKS IN THE QUANTUM-CHROMODYNAMICAL PLASMA

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Abstract

It is suggested that spin-0, color-antitriplet diquarks might occur as a component in the quantum-chromodynamical plasma. Such diquarks would be expected to be favored by Bose statistics at high densities. By treating the plasma as a relativistic ideal gas, it is seen that diquarks indeed carry a large fraction of the total baryon number. An interesting effect is that Bose-Einstein condensation, generally considered to be related chiefly to the low-temperature phenomena of superfluidity and superconductivity, seems to manifest in the QCD plasma at high densities, i.e. high temperatures.

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Currently, much theoretical and experimental effort is being devoted to the study of QCD matter at finite temperature[1]. It is fairly well established that at high temperatures and/or high baryon-number densities, confinement is effectively disabled and chiral symmetry is restored [2]. For systems with vanishing baryon-number density, Monte Carlo simulations of statistical QCD have shown that these transitions occur at the same point [3]. In the case of finite baryon number, the situation is less clear. It has been argued, however, taking instanton effects into account [4] and using finite-temperature QCD sum rules [5] that chiral symmetry restoration cannot precede deconfinement. It is thus possible that there exist three phases of QCD matter : the normal hadronic phase at low temperatures and densities, a plasma phase with deconfined massless quarks, and an intermediate "constituent quark" plasma phase with deconfined massive quarks and massless pions as Goldstone bosons [6]. The chirally symmetric plasma phase, which was universally prevalent during the first microseconds after the big bang and currently is subject to experimental efforts to recreate in relativistic nucleus-nucleus collisions, is generally considered to include light quarks and antiquarks together with gluons as colored components.

However, as has been suggested by several authors [7], there is some evidence that there might exist a bound state of two quarks, a diquark, which could be relatively pointlike at momentum transfers $Q^2 \leq 10\ \mathrm{GeV}^2$. Such spin - 0, color - $\mathbf{3}^*$ objects should, if they exist, also occur as a component in a QCD plasma [8], where they would be expected to be favored by Bose statistics at high densities. Of course, at very high densities, characterised by interquark distances less than $(10\ \mathrm{GeV}^2)^{-1/2}$, diquarks lose their identity and dissolve into quarks. In the intermediate phase, if it exists, diquarks would be expected to be kinematically favored because of constituent mass effects. In the following, for the purpose of illustration of these ideas, we will estimate, using a simple statistical approach, the thermodynamic properties of a plasma including diquarks, and in particular the relative abundance of diquarks.

We will model the chirally symmetric plasma as a relativistic gas of gluons, quarks, antiquarks, diquarks and antidiquarks, and, assuming thermal and chemical equilibrium, use statistical thermodynamics to calculate the properties.

For a statistical ensemble, the grand partition function is

$$\Xi = \mathrm{Tr}\left[\exp\frac{\mu\mathbf{N}-\mathbf{H}}{T}\right], \tag{1}$$

where **H** is the Hamiltonian, **N** the particle-number operator, μ the chemical potential and T the temperature. We always use natural units $\hbar = c = k = 1$.

We will be interested in the high-density regime, and use the free-particle (ideal gas) approximation as suggested by the asymptotic freedom of QCD. The interactions are accounted for [6] simply by introducing a constant energy density B_b and pressure $-B_b$ of the perturbative vacuum, as in the MIT bag model. In this approximation, the trace can easily be performed, in the particle number representation, to give

$$\ln \Xi = \mp \sum_{\mathbf{k}} \ln \left(1 \mp \exp \frac{\mu - E_{\mathbf{k}}}{T} \right), \tag{2}$$

where $E_{\mathbf{k}}$ is the one-particle energy. The upper sign is applicable to bosons and the lower to fermions. Assuming the quantum states $\{\mathbf{k}\}$ to be sufficiently dense [9] , we can approximate the sum by an integral to get (after partial integration)

$$\ln \Xi = \frac{1}{T} \int \sigma(E) \left[\exp \frac{E - \mu}{T} \mp 1 \right]^{-1} \mathrm{d}E. \tag{3}$$

This holds for each component i in the plasma, so the total partition function is

$$\Xi = \prod_i \Xi_i, \tag{4}$$

and the thermodynamic potential is

$$T \ln \Xi = \sum_i T \ln \Xi_i. \tag{5}$$

Thus, we have contributions to the thermodynamic potential from particle type i :

$$T \ln \Xi_i = \int \sigma_i(E) \left[\exp \frac{E - \mu_i}{T} \mp 1 \right]^{-1} \mathrm{d}E, \tag{6}$$

where $\sigma_i(E)$ is the integrated density of one-particle states :

$$\sigma_i(E) = \eta_i \int\limits_{E_i<E} \frac{V\,\mathrm{d}^3p}{(2\pi)^3} = \frac{\eta_i}{6\pi^2} V \left(E^2 - m_i{}^2\right)^{3/2}, \tag{7}$$

since the one-particle energy $E_i = (p^2 + m_i^2)^{1/2}$, and the state density is

$$\rho_i(E) = \frac{\mathrm{d}}{\mathrm{d}E}\sigma_i(E) = \frac{\eta_i}{2\pi^2} V\, E \left(E^2 - m_i{}^2\right)^{1/2}. \tag{8}$$

From

$$\mathrm{d}(T \ln \Xi_i) = S_i\,\mathrm{d}T + P_i\,\mathrm{d}V + N_i\,\mathrm{d}\mu_i, \tag{9}$$

we get

$$P_i = \frac{\eta_i}{6\pi^2} \int\limits_{m_i}^{\infty} \left(E^2 - m_i{}^2\right)^{3/2} \left[\exp\frac{E-\mu_i}{T} \mp 1\right]^{-1} \mathrm{d}E, \tag{10}$$

and, after partial integrations,

$$n_i = \frac{N_i}{V} = \frac{\eta_i}{2\pi^2} \int\limits_{m_i}^{\infty} E \left(E^2 - m_i{}^2\right)^{1/2} \left[\exp\frac{E-\mu_i}{T} \mp 1\right]^{-1} \mathrm{d}E, \tag{11}$$

$$s_i = \frac{S_i}{V} = \frac{\eta_i}{6\pi^2}\frac{1}{T} \int\limits_{m_i}^{\infty} \left(4E^2 - 3E\mu_i - m_i{}^2\right) \left(E^2 - m_i{}^2\right)^{1/2} \left[\exp\frac{E-\mu_i}{T} \mp 1\right]^{-1} \mathrm{d}E. \tag{12}$$

For the energy density, we get

$$\varepsilon_i = \frac{\eta_i}{2\pi^2} \int\limits_{m_i}^{\infty} E^2 \left(E^2 - m_i{}^2\right)^{1/2} \left[\exp\frac{E-\mu_i}{T} \mp 1\right]^{-1} \mathrm{d}E. \tag{13}$$

We see that

$$s_i = \frac{1}{T}\left(P_i + \varepsilon_i - \mu_i n_i\right). \tag{14}$$

Now, consider the plasma to contain the components

$$i \in \left\{u, \overline{u}, d, \overline{d}, s, \overline{s}, D, \overline{D}, g\right\}, \tag{15}$$

where D denotes the *(ud)* diquark.

We get the degeneracy factors η_i from the numbers of spin and color states :

$$\eta_f = \eta_{\overline{f}} = 2 \times 3 = 6, \tag{16}$$

$$\eta_D = \eta_{\overline{D}} = 3, \tag{17}$$

$$\eta_g = 2 \times 8 = 16. \tag{18}$$

Our assumption of chemical equilibrium leads to the following relations for the chemical potentials :

$$\mu_D = \mu_u + \mu_d, \tag{19}$$

$$\mu_g = 0, \tag{20}$$

and, for each quark flavor f :

$$\mu_{\overline{f}} = -\mu_f. \tag{21}$$

Assuming the plasma to contain no net strangeness and to be isoscalar, as would be the case in the forward rapidity region in the current CERN and BNL experimental programmes, with ^{16}O, ^{32}S and, possibly, ^{40}Ca beams [10], we get

$$\mu_u = \mu_d \equiv \mu, \tag{22}$$

$$\mu_s = \mu_{\bar{s}} = 0, \tag{23}$$

$$\mu_D = 2\mu. \tag{24}$$

Of course, we have for a plasma with positive baryon number density

$$\mu > 0. \tag{25}$$

Knowing the masses m_i we can now compute the thermodynamic properties and the abundances of the components of the plasma, knowing that the total net baryon number is conserved ($= B$) ; all as functions of T and μ .

We use massless u and d quarks, and set $m_D = m_s = 225$ MeV ; values we got from an analysis of baryon and kaon production data from e^+e^- annihilation [11] . Introducing a small finite mass of the order of 10 MeV to the lightest quarks does not appreciably alter the results.

Gluons are massless and ultrarelativistic, with zero chemical potential, and the integrals can be calculated exactly to give

$$P_g = \frac{1}{3}\varepsilon_g = \frac{8}{45}\pi^2 T^4, \tag{26}$$

$$s_g = \frac{32}{45}\pi^2 T^3, \tag{27}$$

$$n_g = \frac{16}{\pi^2} T^3 \varsigma(3) \approx 1.95\, T^3. \tag{28}$$

For each massless quark flavor $q = u$ or d, the contributions to the pressure, as well as to the baryon-number, energy and entropy densities, can, miraculously, be calculated analytically. We get

$$n_q - n_{\bar{q}} = \mu T^2 + \frac{1}{\pi^2}\mu^3, \tag{29}$$

$$P_q + P_{\bar{q}} = \frac{1}{3}(\varepsilon_q + \varepsilon_{\bar{q}}) = \frac{7}{60}\pi^2 T^4 + \frac{1}{2}\mu^2 T^2 + \frac{1}{4\pi^2}\mu^4, \tag{30}$$

$$s_q + s_{\bar{q}} = \frac{7}{15}\pi^2 T^3 + \mu^2 T. \tag{31}$$

However, as is also the case for the massive plasma components, the separate quark and antiquark contributions cannot be evaluated exactly, so we have calculated all integrals (except for the gluons) numerically and used these analytic results merely as a consistency check.

To get the properties of the plasma solely as functions of the temperature T, we need an additional relation; an evolution equation governing the cooling plasma. We shall assume that cooling through hydrodynamic expansion predominates over cooling through radiation, which we neglect. Thus, we assume that the plasma cools and expands isentropically [12] :

$$\mathrm{d}S = 0. \tag{32}$$

An interesting effect arises due to the inclusion of massive bosons in equilibrium with the quarks in the plasma. As the Bose distribution cannot take on a negative value, we get an upper bound on the boson chemical potential :

$$\mu_D \leq m_D, \tag{33}$$

and, consequently,

$$\mu \leq \frac{m_D}{2}. \tag{34}$$

This does not limit the possible baryon number density, since at $\mu_D = m_D$ a finite fraction of the baryon-number carrying massive bosons can condense in the ground state $p = 0$. These correspond to the first term in (2), and are not counted by the integral in (3). This is an example of the well-known text-book phenomenon of Bose-Einstein condensation [13], traditionally considered to be related to the λ - transition of helium and to Cooper-pair formation [14]. Such condensation is indeed seen to occur at high densities, i.e. high temperatures, in the quantum-chromodynamical plasma. It is interesting to note that this phenomenon, which is assumed to be of relevance to the low-temperature phenomena of superfluidity and superconductivity, also seems to manifest in a QCD plasma at temperatures above 10^{12} K.

At first sight it might seem counter-intuitive that condensation takes place *above* a critical temperature, but this is naturally caused by the increasing density. In the classical idealised case of helium, it is easy to see that as the temperature is lowered at *constant density*, the chemical potential increases and reaches saturation. In this case of *constant entropy*, the chemical potential is seen to increase with temperature.

We have, somewhat arbitrarily, chosen to present results for the case of a constant entropy of 20 units per baryon. This choice might not be too unrealistic [15]. In Fig. 1, the evolution of the plasma in the μ - T plane is shown. It is seen that μ is saturated at its maximum value $m_D/2$ until the temperature has dropped to $T_e = 190$ MeV. After this point, μ drops fairly linearly until chiral symmetry is broken.

Fig. 2 shows the equilibrium abundances of the plasma components. We see that for temperatures above 140 MeV, diquarks are more abundant than quarks. This is due to the fact that at high temperatures, i.e. high densities, quantum-statistical effects start to outweigh the kinematical suppression due to rest mass. In Fig. 3, we display a log-log plot of the total baryon-number density, normalised to that of nuclear matter, versus temperature. It is seen that the behavior is very close to a pure power law. The best fit exponent is 2.98, but $n_B \sim T^3$ also gives an excellent fit, so in this respect we get the same behavior as for a pure massless quark-gluon plasma. The total energy density is shown in Fig. 4 as a function of temperature. For an ideal massless quark-gluon plasma one gets $\varepsilon \sim T^4$, but here we see a somewhat different behavior. The power-law fit is less perfect, and the best-fit exponent is 3.66. This slower rise will, as we shall see, be explained by the increasing fraction of ground-state diquarks.

In Figs. 5a and 5b, we show the fraction of the total baryon number of the plasma carried by the net diquark component. As is seen, this fraction rises with temperature and density. The derivative of this function shows a discontinuity at the temperature $T_e = 190$ MeV, corresponding to a density $(n_B)_e = 6.5\,(n_B)_0$. This is the point beyond which Bose-Einstein condensation in momentum space occurs. The extent to which the diquarks condense is shown in Figs. 6a and 6b, as functions of temperature and density, respectively. We see a sharp initial rise in this fraction, so that only slightly beyond the "critical" limit a significant portion of diquarks are in the same quantum state. Such a "superfluid" plasma component would be likely to remain even in a refined picture where interactions are explicitly taken into account. It might be speculated that collective and coherent phenomena should be drastically enhanced in the plasma in this region. One could, for instance, be more optimistic regarding the possibility of thermalisation. More theoretical work along these lines is called for.

If the condensate should prevail down to the hadronisation regime, one would expect a strong enhancement of the production of multiquark states, such as three-diquark dibaryons, since the diquarks in the condensate all have zero relative velocity, apart from finite-size effects.

It should be noted that we have neglected the component of strange diquarks, *(us)* and *(ds)*, which should be expected to contribute at high temperatures. However, these heavier diquarks, which are perhaps even more pointlike than the *(ud)* diquarks [16], cannot form a condensate, since their equilibrium chemical potential is μ, which cannot exceed $m_D/2$, and $m_{(us)} > m_D$.

All results are given for the case of a constant entropy / baryon of 20 units. One might reasonably ask how sensitive the analysis is to the precise value of this parameter. The phenomenon of Bose-Einstein condensation of the diquark component occurs above a certain temperature, T_e, and above an associated density, $(n_B)_e$. In Figs. 7a and 7b the variation of this condensation temperature and condensation density with the input parameter S/B is shown. It is seen that T_e to a remarkable extent is directly proportional to S/B over the range studied, and that the associated quantity $(n_B)_e$ is very well described by a power-law behavior. The best-fit exponent is 1.96, but $(n_B)_e \sim (S/B)^2$ is also in excellent agreement with the calculated points.

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Footnotes and references

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Figure captions

Fig. 1 The evolution of the plasma in the $\mu - T$ plane for a constant entropy per baryon of $S/B = 20$.

Fig. 2 The equilibrium abundances of the plasma components for $S/B = 20$, as functions of temperature. Quarks denote n_u or n_d, similarly for antiquarks.

Fig. 3 The baryon-number density, normalised to that of nuclear matter $(n_B)_0 \approx 0.17\ \mathrm{fm}^{-3}$, as a function of temperature for $S/B = 20$

Fig. 4 The total plasma energy density for $S/B = 20$, as a function of temperature.

Fig. 5 The fraction of the total baryon number of the plasma carried by diquarks for $S/B = 20$, versus temperature (5a) and normalised baryon-number density (5b).

Fig. 6 The fraction of diquarks in the plasma that are subject to Bose-Einstein condensation for $S/B = 20$, as functions of temperature (6a) and normalised density (6b).

Fig. 7 The variation of the point at which Bose-Einstein condensation of the diquarks in the plasma starts to occur with the input parameter S/B. In Fig. 7a is plotted the condensation temperature T_e, and in Fig. 7b the associated (normalised) condensation density $(n_B)_e/(n_B)_0$, versus S/B.

Fig. 1

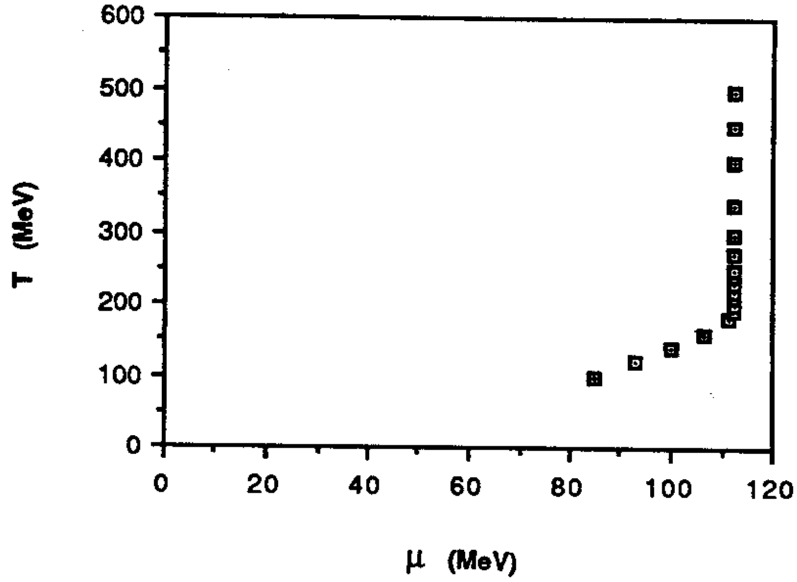

600
500
400
300
200
100
0
T (MeV)
0
20
40
60
80
100
120
μ (MeV)

Fig. 2

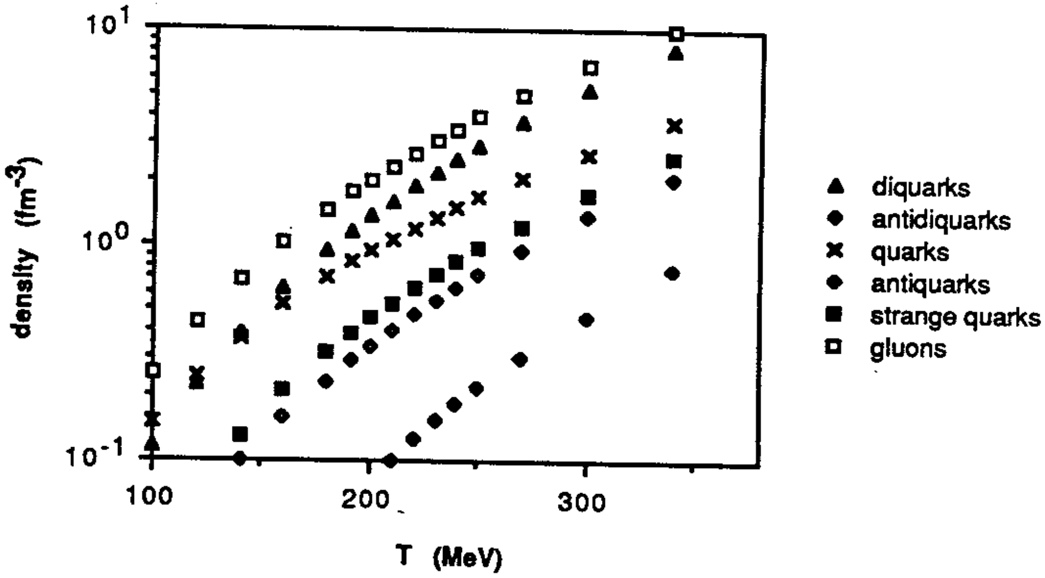
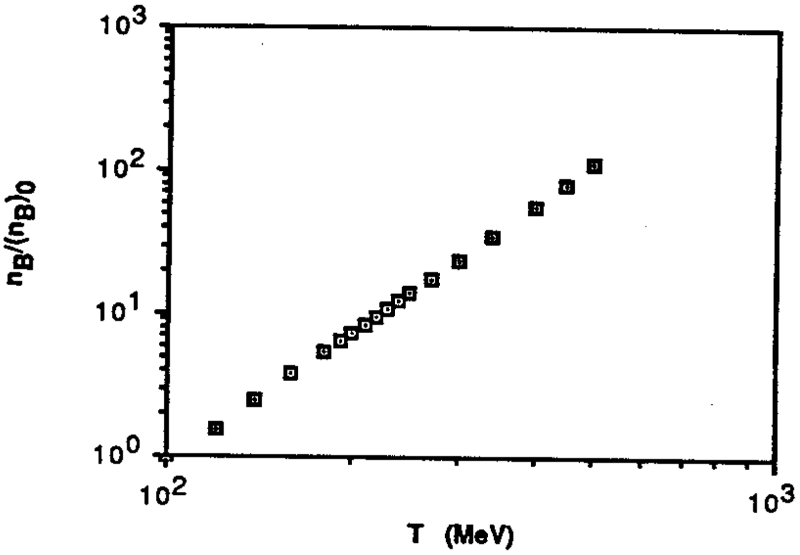
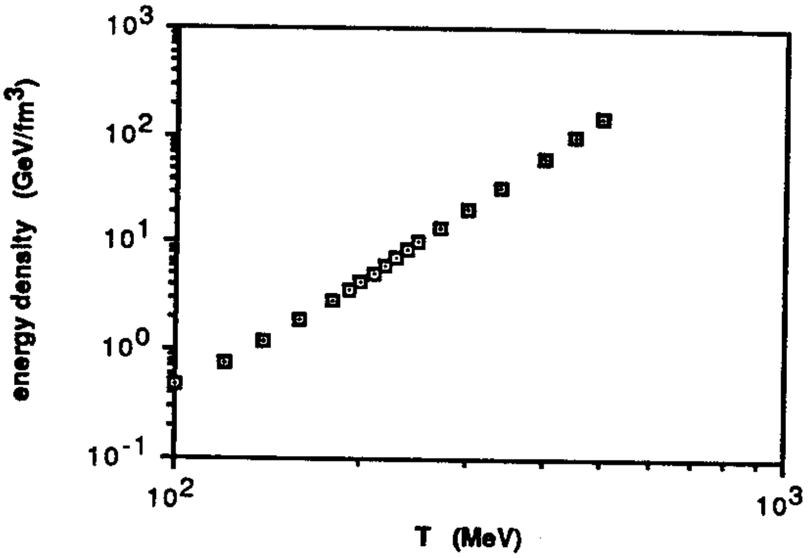

10^1
10^0
10^{-1}
density (fm^{-3})
100
200
300
T (MeV)
diquarks
antidiquarks
quarks
antiquarks
strange quarks
gluons

Fig. 3



10^3
10^2
10^1
10^0
$n_B/(n_B)_0$
10^2
10^3
T (MeV)

Fig. 4



10^3
10^2
10^1
10^0
10^{-1}
energy density (GeV/fm^3)
10^2
10^3
T (MeV)

Fig. 5a

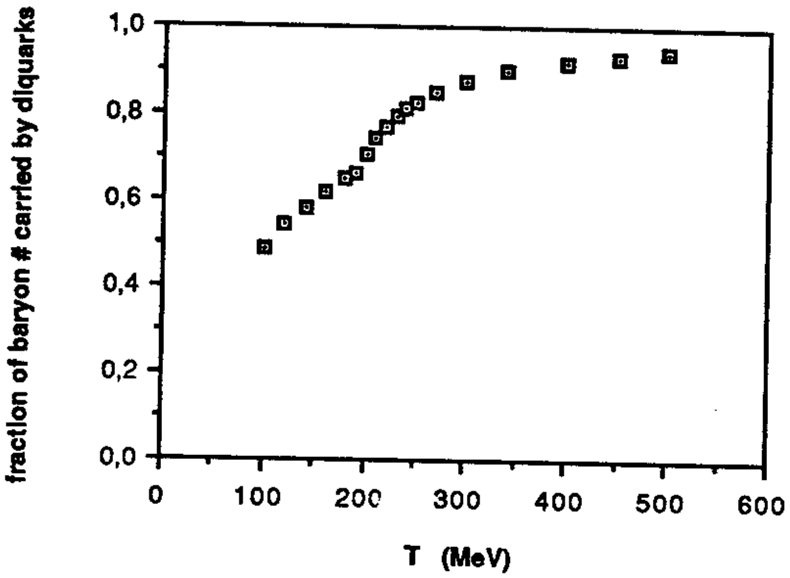

fraction of baryon # carried by diquarks
1,0
0,8
0,6
0,4
0,2
0,0
0
100
200
300
400
500
600
T (MeV)

Fig. 5b

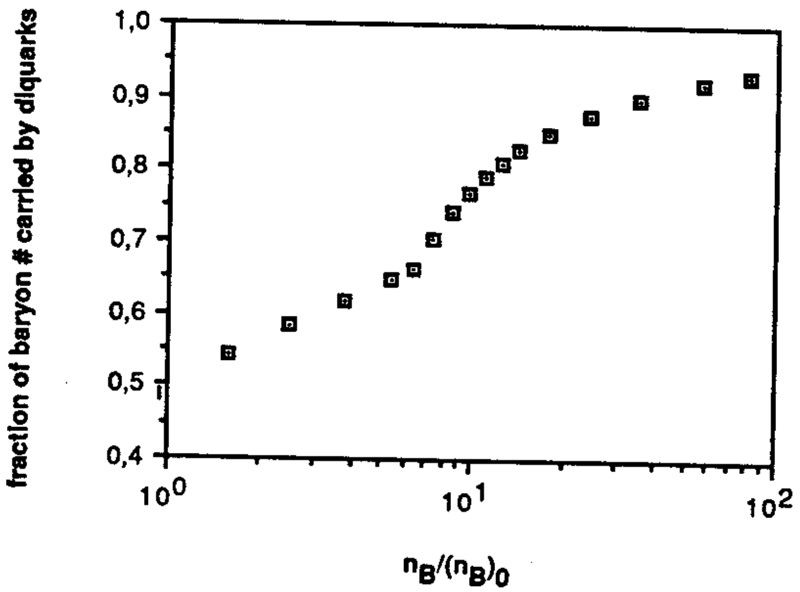
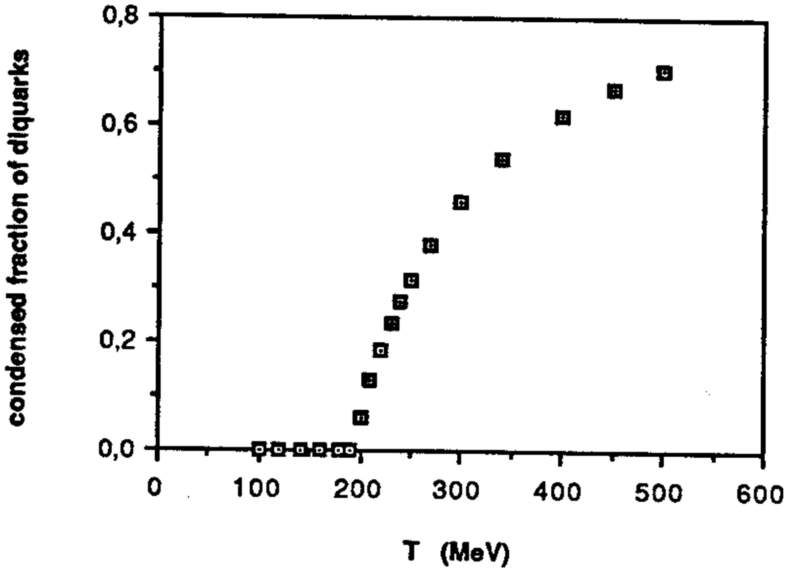
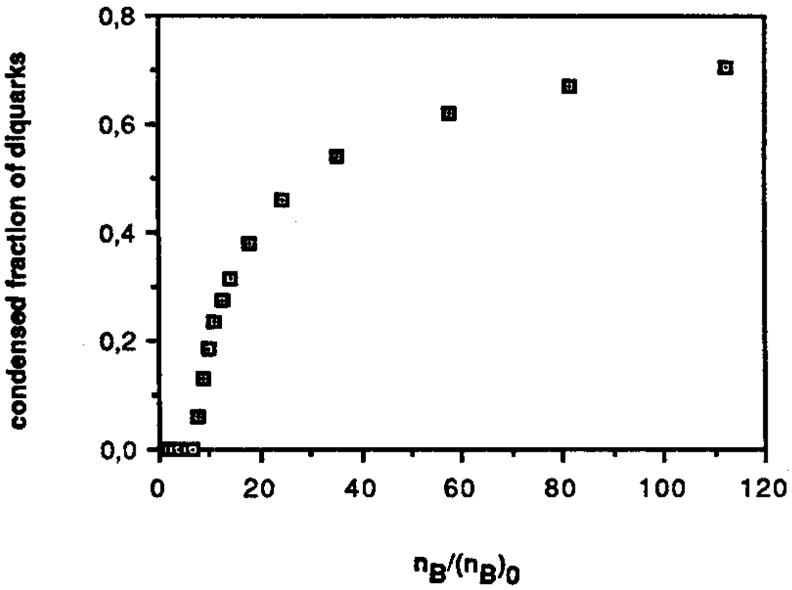

fraction of baryon # carried by diquarks
1,0
0,9
0,8
0,7
0,6
0,5
0,4
10^0
10^1
10^2
$n_B/(n_B)_0$

Fig. 6a



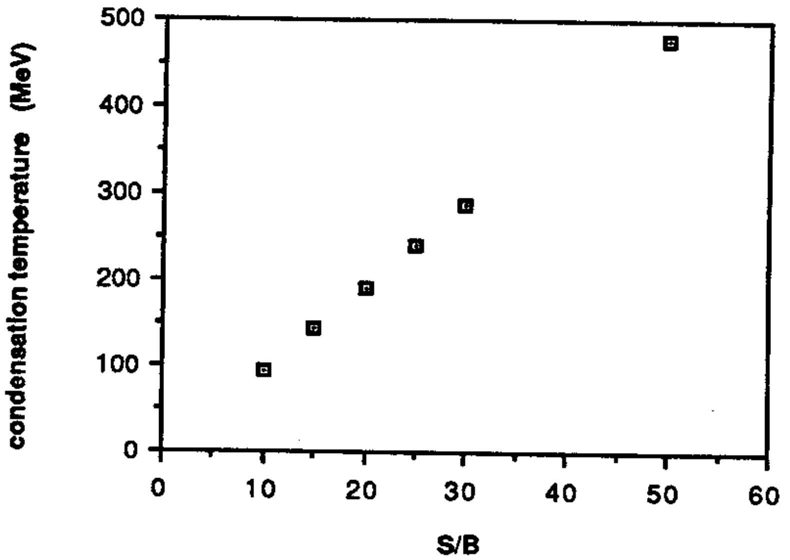
condensed fraction of diquarks
0,8
0,6
0,4
0,2
0,0
0
100
200
300
400
500
600
T (MeV)

Fig. 6b



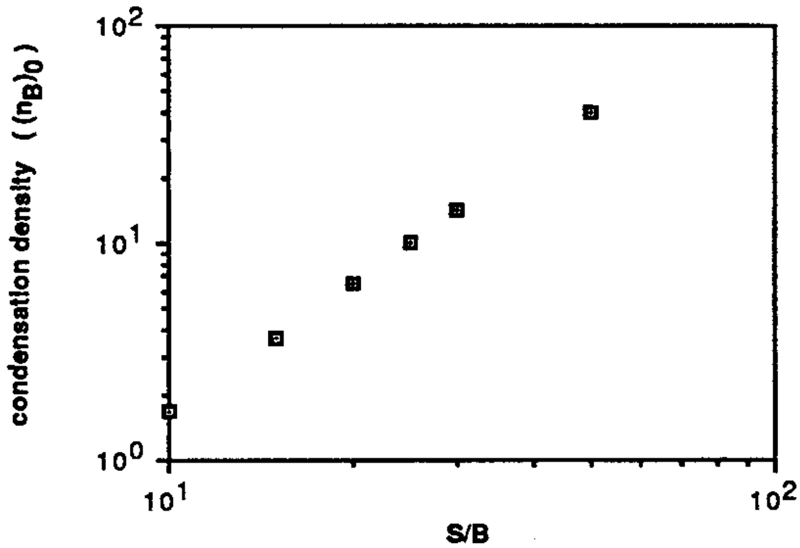
condensed fraction of diquarks
0,8
0,6
0,4
0,2
0,0
0
20
40
60
80
100
120
$n_B/(n_B)_0$

Fig. 7a



condensation temperature (MeV)
500
400
300
200
100
0
0
10
20
30
40
50
60
S/B

Fig. 7b



condensation density $(\langle n_B \rangle_0)$
10^2
10^1
10^0
10^1
10^2
S/B